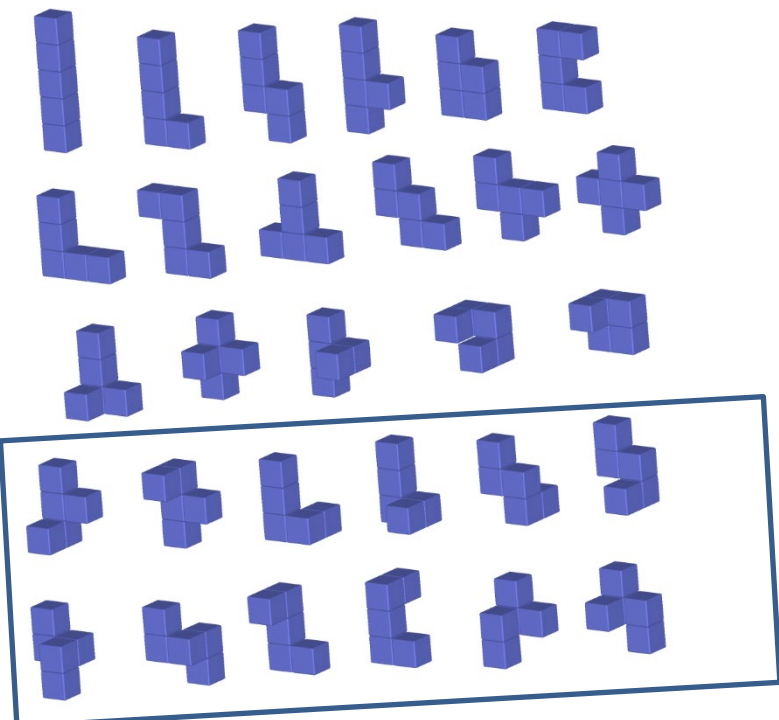# Senior Challenge '24 Solutions

1. Pegasus	
It takes 63.75 minutes to do Deeside to Uffington giving a start time of 11:36:15, which is 1136 to the nearest minute. Leaving Uffington at 1300, the journey to Caen takes 86.25 minutes, arriving at 14:26:15, so 1426 to the nearest minute.	4 marks: (1 mark for 1136, 1 mark for reasoning, 1 mark for 1426, 1 mark for reasoning) Special Case: Award a maximum of 2 marks for correct reasoning, but incorrect rounding.
2. Triangulum	
The key thing to remember here is that in any triangle the sum of two sides must always be greater than the third side. For 2 and 4 cm sides, the only possibilities are: (2, 3, 4), (2, 4, 4) and (2, 4, 5). If the largest side is 5cm then there are 22 different triangles: (1, 1, 1), (1, 2, 2), (1, 3, 3), (1, 4, 4), (1, 5, 5), (2, 2, 2), (2, 2, 3), (2, 3, 3), (2, 3, 4), (2, 4, 4), (2, 4, 5), (2, 5, 5), (3, 3, 3), (3, 3, 4), (3, 3, 5), (3, 4, 4), (3, 4, 5), (3, 5, 5), (4, 4, 4), (4, 4, 5), (4, 5, 5), (5, 5, 5).	5 marks: (1 mark for 3, 1 mark for listing (2, 4, X), 1 mark for 22, 3 marks for listing all (X, Y, Z), 1 mark for stating X+Y>Z) The marks for listing solutions can be assumed if a good enough explanation is provided.
3. Eridanus	
6r is the river's flow in mph and 6V is the boat's speed in still water in mph. Lucie goes upstream for 10 minutes, which is V – r miles. The bottle meanwhile has drifted downstream by r. Lucie and the bottle are thus separated by V – r + r = V miles when she turns around. She travels downstream at (6V + 6r) mph. Relative to the bottle, therefore, she travels at 6V mph, so it takes her another 10 minutes to close the gap of V miles and catch the bottle. Since the bottle has now travelled 2 miles in twenty minutes, the river flows at 6mph. This can be verified by noting that the boat travels V + r miles downstream in 10 minutes, but had already travelled V – r upstream in 10 minutes, so it is (V + r) – (V – r) = 2r miles downstream from the starting point after 20 minutes. The bottle also travels 2r miles downstream in 20 minutes, so they're at the same point after 20 minutes. That means 2r = 2, so 6r = 6, which is 6mph.	4 marks: (1 for V – r for upstream, 1 for V + r for downstream, 1 for 2r = 2 miles or 2 miles in 20 minutes for bottle, 1 for correct answer of 6mph)

4. Camelopardalis, Apus and Monoceros

G= number of giraffes, U = number of unicorns, B = number of birds of paradise Equations: Height: $7.5G+2U+0.25B=50$, Number: $G+U+B=100$, Cost: $98G+42U+6B=1000$ G= 2, U = 6, B = 92 So, **2 Giraffes, 6 Unicorns and 92 Birds of Paradise**	6 marks: (1 mark for each equation, 1 mark for each correct answer) Award all 6 marks for correct answers with valid alternative reasoning.

5. Gemini

29 ways of arranging 5 cubes, including **six pairs of twins**. 	6 marks: (1 mark for 29, 1 mark for six pairs of twins, 3 marks for drawing them all out, 1 mark for identifying pairs of twins, as shown in the box) 29 diagrams – 3 marks 27, 28, 30, 21 – 2 marks 20+ diagrams – 1 mark

6. Canis Major and Canis Minor

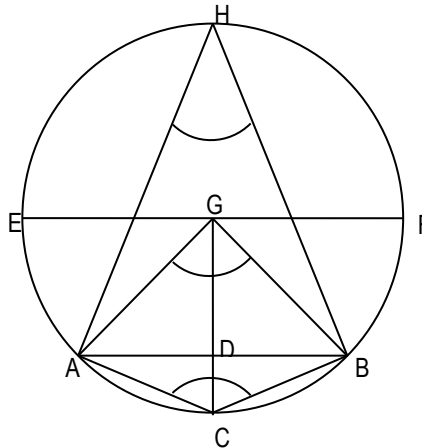|	
When Major is finished with a piece of carpet the number of pieces has increased by three (one piece becomes four) and when Minor is finished with a piece the number has increased by six (one piece becomes seven). So, however many times either dog does his or her thing, the number of pieces at the end will have the form 1 + a multiple of 3. Now, 2024 = (674 × 3) + 2, which is 2 more than a multiple of 3. So, 2024 cannot be the full tally of carpet pieces: there must be at least two more hidden away somewhere.	6 marks: (1 mark for 1 to 4 and 1 to 7, 1 mark for increase by 3 and increase by 6, 1 mark for 1 + 3n, 1 mark for 2024 isn't all of them 2 marks for reasoning)

7. Sagittarius

This diagram shows a cross-section through the sphere. Points A and B are on either edge of the crater, point C is at its base. Point D is the midpoint of line AB. Line EF is a diameter of the circle. Point G marks the centre of the circle. Point H is a point on the edge of the circle (drawn centrally for elegance) The distance from A to B is 240mm, CD is 60mm		7 marks: (1 mark for each calculation in the first method OR 1 mark for GB & GC are radii, 1 mark for GB = R, 1 mark for GD = R – 60, 1 mark for applying Pythagoras, 2 marks for the calculation, 1 mark for R = 150mm) NB There are several other valid ways to solve this. Award 7 marks for any method with working that produces the right answer. Special Case: Allow up to 5 marks if calculations are correct, but rounding error has pushed result off from 150mm.

<ACD is found using $tan^{-1}\frac{120}{60} \approx$ **63.4°**

This makes <ACB ≈**126.9°**

Then <AHB=180-126.9≈**53.1**°

Then <AGB=2×<AHB≈**106.2**°

Then <DGB=½<AGB≈**53.1°**

ΔDGB has a right angle at D, so the length GD is found by using
$$\tan(< DGB) = \frac{120}{GD}$$
This rearranges to give us
$$GD = \frac{120}{\tan(<DGB)} \approx \frac{120}{\tan(53.1)} = \mathbf{90mm}$$
Thus, the radius of the sphere is GC=90+60=**150mm**.

An alternative approach is to observe that GB and GC are both radii. Denoting the radius as R, we have GB = R and GD = R – 60.

Since D is the midpoint of AB, then DB = 120mm.

ΔDGB has a right angle at D, so, by Pythagoras:

R^2 = (R – 60)^2 + 120^2 = R^2 – 120R + 3600 + 14,400.

This simplifies to 120R = 18,000, so **R = 150mm**.